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Regimes of Oscillation in a Linear Array of Vortices

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The instabilities of a linear array of vortices are studied experimentally, for two values of the thickness of the fluid layer. In the first case (thickness of 2 mm), the system is found to behave like a chain of coupled oscillators. New experimental results obtained for a thickness of 3 mm are described. The qualitatively different behavior found in the latter case may be the signature of the influence of a second-neighbor coupling.

KEY WORDS: Instabilities; chaos; extended systems; vortices.

The transition to spatiotemporal chaos is a problem of great current interest. Because of the large number of possible routes leading to weak turbulence, many studies have been devoted to systems of low dimension (see, e.g., refs. 1). In this report, we present recent results obtained with a system whose spatiotemporal dynamics is essentially one dimensional in space.

The experimental system which we consider herein is a linear array of vortices. It is similar to that used in previous studies.[(2)] The cell is an open rectangular container, 350 mm long, 50 mm high, and 40 mm wide, machined out of perpex or PVC. The working fluid, which is a normal solution of sulfuric acid, is placed in a groove, 300 mm long, 20 mm large, machined in the bottom plate of the cell. Experiments are carried out for two values of the depth of the groove b: $b = 2$ mm (experiment A) and $b = 3$ mm (experiment B). Throughout the experiment, the thickness of the fluid layer is maintained at a value equal to b in order to keep the free surface flat when the fluid is at rest. Just below the fluid layer, a line of permanent magnets is formed; each individual magnet is a samarium cobalt

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parallelepiped, of dimensions $5 \times 8 \times 3$ mm. They are put together to form a line of alternating poles. The vertical component of the resulting magnetic field, within the liquid layer, is a periodic function—close to a sinusoidal function—of the coordinate along the lattice, with a zero mean value, and an amplitude of 0.3 T. A steady electric current I is driven longitudinally through the electrolyte; it interacts with the magnetic field to force the flow.

For quantitative analysis, we use a shadowgraph technique, based on the fact that the free surface of the liquid is deflected by the rotation of the vortices. We thus form the image of the perturbed surface of the fluid by using a system of two confocal lenses, characterized by lateral magnifications ranging from 1/15 to 1/50. This method allows for visualizing the separatrices between vortices as light lines. We further digitize the image and track the position of such lines. In the time-dependent regimes, the corresponding ratio of signal to noise is about 40 dB.

At low currents, the basic state of flow is a linear array of counter-rotating vortices of sizes equal to that of an individual magnet. As the electric current is increased, the vortices cease to have a uniform size along the lattice axis: half of them become larger at the expense of the others, which decrease.[2] As I is increased further, the small vortices disappear, leaving the system composed only of stationary corotating vortices, with a size twice as large as that in the basic state. We denote this state by "state +". This state will further become unstable as the electrical current is increased.[2] The dynamics of the system is studied by measuring the oscillations of the positions of the separatrices along the lattice.

In this report, we first summarize the results of experiment A (i.e., for $b = 2$ mm) and further report new results obtained for experiment B (i.e., for $b = 3$ mm). The reader is referred to ref. 2 for a more complete description of experiment A.

In experiment A, we find that the bifurcation from state + to the time-dependent state is a supercritical Hopf bifurcation. The spatial structure of the oscillating mode is in the form of an optical mode, i.e., the separatrices oscillate out of phase with their nearest neighbor, with a slowly varying amplitude along the lattice. The destabilization of this regime corresponds to the onset of an instability that we call the "short-wavelength instability." In the new regime, the amplitudes of oscillations are no longer homogeneous along the lattice, but show significant variations from one oscillator to the other. The corresponding wavelength is thus twice the lattice mesh. There is a third bifurcation as I is further increased, whose characteristics depend on the length of the system. For moderately large lattices, the new state is a quasiperiodic regime, with two frequencies, while for systems of larger sizes, the new state is a chaotic regime. These transitions are summarized in the phase diagram of Fig. 1a.

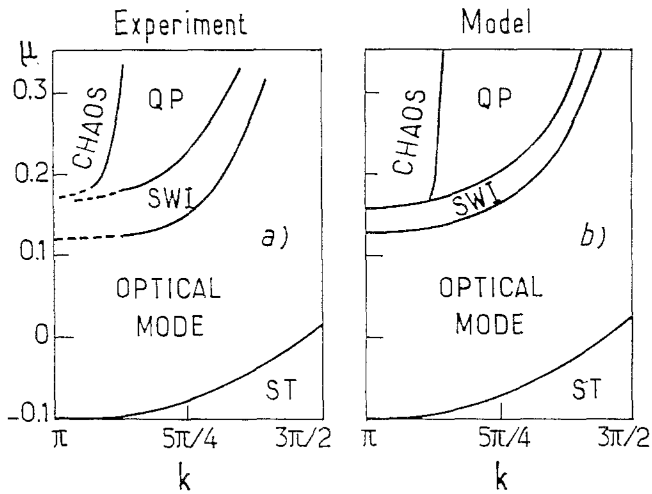

Fig. 1. Phase diagrams of the system for experiment A: (a) experiment; (b) Eq. (1), with the coefficients determined experimentally; $\mu = (I - I_c)/I_c$, where $I_c \approx 12.1$ mA. The abbreviations are: ST, stationary; SWI, short wavelength instability; QP, quasiperiodic state.

Owing to the particular geometry of the experiment, it is tempting to compare the system to a linear chain of coupled oscillators. In the case that we consider, such oscillators are necessarily supercritical and a simple model can be written down under a nearest neighbor coupling assumption, and using the symmetries of the system. We thus have

$$\frac{\partial W_n}{\partial t} = \mu(1 + ic_0)\, W_n - (1 + ic_2)\, W_n\, |W_n|^2 + \varepsilon(1 + ic_1)(W_{n-1} + W_{n+1})$$
$$- (c_3 + ic_4)\, W_n(|W_{n-1}|^2 + |W_{n+1}|^2) \qquad (1)$$

in which W_n is the complex amplitude of the nth oscillator, t is the dimensionless time, and μ, ε, c_0, c_1, c_2, c_3, and c_4 are real coefficients. Models similar to Eq. (1), but restricted to linear couplings, have been studied by Kuramoto.[3]

According to Eq. (1), the most unstable mode is either an acoustical mode (for $\varepsilon > 0$), where all the oscillators are in phase, or an optical mode (for $\varepsilon < 0$), where each oscillator is out of phase with respect to its nearest neighbors. One further finds that, depending on the values of the coefficients, such regimes are unstable either to long-wavelength perturbations (i.e., Benjamin–Feir instabilities), or to short-wavelength perturbations. In the latter case, the onset of instability is located at finite distance from the primary instability point; the new state is monoperiodic in time and its spatial structure is such that half of the amplitudes of oscillation are larger than those of their immediate neighbor; the corresponding wavelength is thus exactly twice the lattice mesh. Clearly, the structure of this mode is very close to that observed in the experiment (see the SWI introduced above). Actually, because of the richness of Eq. (1), one must be more quantitative to conclude that the two instabilities are the same.

It turns out that the experimental situation allows one to determine the coefficients of Eq. (1); for that, we used a strategy somewhat similar to that of a recent study.[4] The separatrices are labeled from 1 to N along the lattice and the order parameter W_n is understood as related to the temporal behavior of the nth separatrix. We thus find the following values for the coefficients of Eq. (1): $c_0 \approx 2.1$, $c_1 \approx -4.3$, $c_2 \approx -0.2$, $c_3 \approx -0.08$, $c_4 \approx -1.2$, $\varepsilon \approx -0.05$, and $\mu = (I - I_c)/I_c$, where $I_c \approx 12.1$ mA. Using these values, one can further determine the nature of the secondary instability of the system. The system is found to be stable to long-wavelength modes, and unstable against short-wavelength perturbations. As mentioned above, the corresponding instability has precisely the form found in the experiment. All these theoretical results are in agreement with a numerical study of Eq. (1), where the lattice is supposed to have perfectly reflecting ends; the phase diagram calculated with the above values of the coefficients is shown in Fig. 1b, and the results are in remarkable agreement with the experiment.

We therefore find a good agreement between the model and the experiment. We have revealed the existence of a small-scale instability—called the SWI—which breaks the same symmetry as the Benjamin–Feir instability, but is short wavelength (twice the lattice mesh) and appears at finite distance from the primary instability point. This instability is related to the discrete nature of our system, and to the existence of nonlinear interactions between the oscillators.

We now describe results obtained on experiment B. The system has been studied for a number of magnets ranging from 4 to 38; this corresponds to a number of corotating vortices varying from 2 to 19, and therefore a number of oscillators ranging from 1 to 18 (we assume here that one can still establish an analogy between this system and a chain of oscillators, as above). In this case, as in experiment A, the first regime of oscillation is monoperiodic and the corresponding bifurcation is clearly supercritical in systems of small sizes; however, as soon as the number of corotating vortices becomes larger than 3, its spatial structure is very different from that observed in experiment A: in particular, in large systems, there is no standing wave at onset, but propagative waves, or counterpropagating waves, depending on the initial conditions and the control parameter; the amplitudes of oscillation are not smooth functions of the lattice coordinate, but show large variations along the system. These features appear on the direct time recordings of Fig. 2, where a fraction of the lattice is represented. In this case, there are 34 magnets, and the system is close to the onset of the oscillation; the regime is monoperiodic at frequency $f = 180$ mHz. The amplitude of oscillation is clearly nonuniform along the lattice: the oscillators are structured into regions where the oscillation has a large amplitude, separated by deep minima of amplitude. Figure 2 typically represents the dynamical state of the system above onset of oscillation. The domains where the oscillators have a significant amplitude are usually composed of one, two, or three elements. There is no clear relation between their size and the control parameter except a tendency to decrease as I is increased. The positions of the minima and maxima of amplitude are dependent on the initial conditions, so that it is difficult to invoke experimental imperfections to account for their existence.

One can propose a rough classification of the types of structures that we observe. They are represented in Fig. 3, which shows typical spatial evolutions of the amplitudes and phases along the lattice. Figure 3a represents a part of the system sustaining a single propagating wave, traveling from left to right, with a wavelength equal to about four times the lattice mesh. In contrast with the simplicity of the spatial structure of the phase, that of the amplitude is more complex; it shows large variations from one oscillator to the other. Another type of structure that we observed

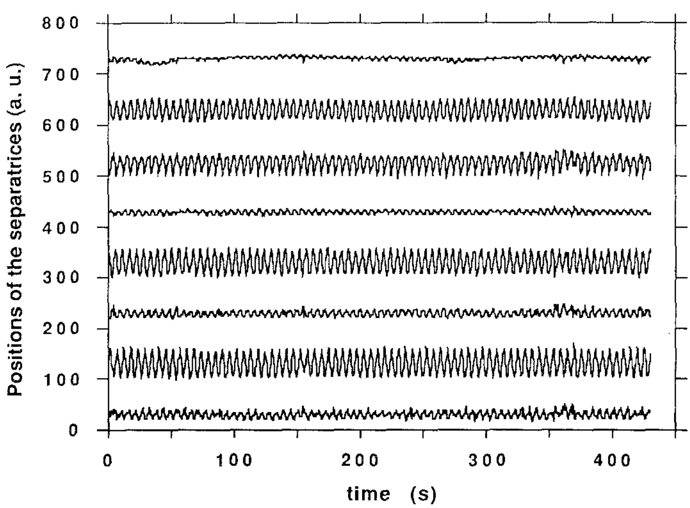

Fig. 2. Direct time recordings of the oscillations of the separatrices in part of a lattice composed of 17 corotating vortices (34 magnets), for $I = 23$ mA and $b = 3$ mm (experiment B). Each record corresponds to the instantaneous position of a separatrix. The regime is monoperiodic.

corresponds to a sink (see Fig. 3b): in this case, there are two counter-propagating waves, separated by a sink. Sudh sinks seem to be associated with a zero of amplitude and a phase shift of π (see Fig. 3b). It is interesting to mention that we observe at most a single sink in the system. As the control parameter is increased, the sink migrates along the lattice, usually toward an extremity. Sinks and deep minima of amplitude divide the system into "domains", as described above. Another type of structure that we have observed is shown in Fig. 3c: within a single "domain", one

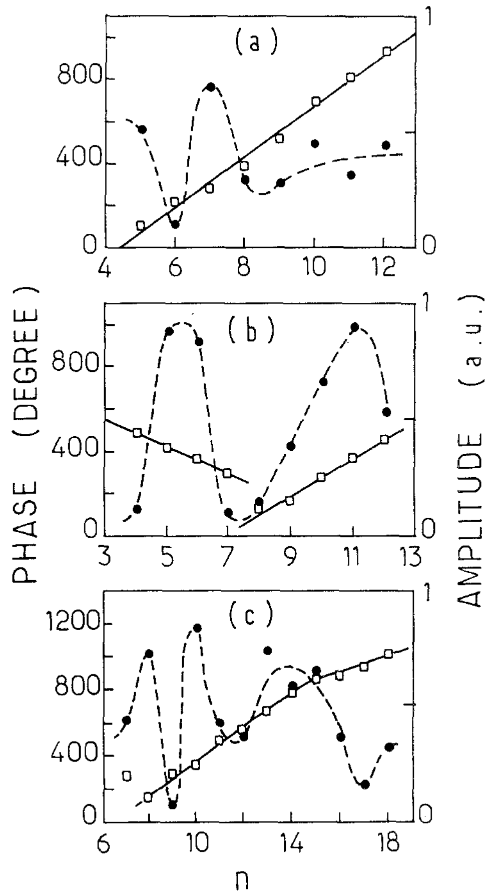

Fig. 3. Typical phase and amplitude spatial evolution in the monoperiodic state of experiment B, for 38 magnets. The black points correspond to the amplitude, and the squares to the phase; dashed and full lines are drawn to guide the eye. Variable n denotes the position of the separatrix along the lattice (labeled from 1 to 18 in this case); (a) traveling wave with large-amplitude variations from one oscillator to the other; (b) presence of a sink; (c) change of wavelength.

can have a shift in the phase velocity of the wave; however, in this case, such a shift is not associated with anything particular concerning the amplitude.

The dynamical state of the system above the oscillation onset is thus characterized by traveling waves, domains, sinks, minima of amplitude, and phase velocity shifts. All such structures are present in a disordered way along the lattice. There is therefore no spatial order in this case. As the control parameter is increased, the system becomes unstable, giving rise to quasiperiodicity and chaos. Figure 4 shows the chaotic state, in the case of 34 magnets, for $I = 23$ mA. In this case, some oscillators oscillate for some period of time, then decrease, further oscillate again, and so on, at random. This is the type of chaotic behavior that we get in this system; it seems very different from that found for experiment A.

The two experiments thus show two strongly different behaviors: in experiment A, for the optical mode regime, the amplitudes of the oscillators are slowly modulated along the lattice. In contrast, in experiment B, one observes small-scale variations of the amplitude at onset, so that the system appears to be composed of clusters of oscillators. It is interesting to note that similar observations have been made for a line of convective cells.[5] One can ask if there still exists a model of coupled oscillators which reproduces the structure of such regimes. An interesting possibility is to include a second-neighbor coupling effect, which may give rise, as in

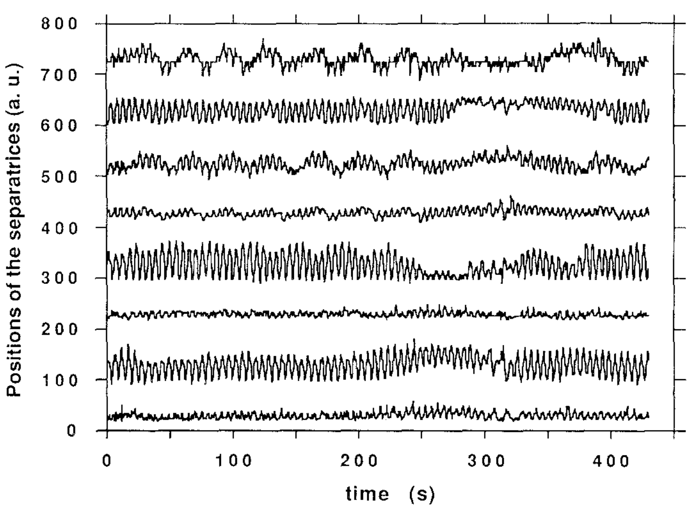

Fig. 4. Direct time recordings of the oscillations of the separatrices in part of a lattice composed of 17 corotating vortices (34 magnets), for $I = 23$ mA and $b = 3$ mm (experiment B). The regime is chaotic.

helimagnetic systems, to a strong modulation of the order parameter along the lattice, in agreement with the experiment. Such a possibility indeed remains to be examined.

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